Superfluid transition in a rotating resonantly-interacting Fermi gas

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(Dated: July 28, 2006)

We study a rotating atomic Fermi gas near a narrow s-wave Feshbach resonance in a uniaxial harmonic trap with frequencies $\Omega_\perp$, Ω_z. Our primary prediction is the upper-critical angular velocity, $\omega_{c2}(\delta, T)$, as a function of temperature T and resonance detuning δ, ranging across the BEC-BCS crossover. The rotation-driven suppression of superfluidity at ω_{c2} is quite distinct in the BCS and BEC regimes, with the former controlled by Cooper-pair depairing and the latter by the dilution of bosonic molecules. At low T and $\Omega_z \ll \Omega_\perp$, in the BCS and crossover regimes of $0 \lesssim \delta \lesssim \delta_c$, ω_{c2} is implicitly given by $\hbar\sqrt{\omega_{c2}^2 + \Omega_\perp^2} \approx 2\Delta\sqrt{\hbar\Omega_\perp/\epsilon_F}$, vanishing as $\omega_{c2} \sim \Omega_\perp(1-\delta/\delta_c)^{1/2}$ near $\delta_c \approx 2\epsilon_F + \frac{\gamma}{2}\epsilon_F \ln(\epsilon_F/\hbar\Omega_\perp)$ (with Δ the BCS gap and γ the resonance width), and extending the bulk result $\hbar\omega_{c2} \approx 2\Delta^2/\epsilon_F$ to a finite number of atoms in a trap. In the BEC regime of $\delta < 0$ we find $\omega_{c2} \to \Omega_\perp^-$, where molecular superfluidity can only be destroyed by large quantum fluctuations associated with comparable boson and vortex densities.

PACS numbers: 03.75.Fi, 03.75.Lm, 05.30.Jp

Recent advances in atomic gases near a Feshbach resonance (FR) have led to the realization of resonantly-paired atomic superfluids [1, 2]. The proximity to a FR allows a tunability of the pairing interaction, thereby permitting unprecedented access to fermionic superfluidity ranging from a weakly-paired BCS regime to a strongly-paired molecular BEC regime.

A fundamental aspect of a superfluid is its non-classical response to an imposed rotation. Unable to exhibit rigid body rotation, a superfluid rotates by nucleating a vortex array with density $n_v = m\omega/\pi\hbar$ set by the rotation rate ω. Although considerable progress has been made in elucidating the properties of rotating superfluids in the BEC regime (with bosonic atoms) [3, 4, 5], considerably less is understood for a resonantly-paired trapped superfluid in the BCS and crossover regimes. Recent spectacular experiments by Zwierlein, *et al.* [6] provide strong motivation for a study of these regimes.

Additional motivation is provided by the relation of a rotating superfluid to a type-II superconductor in a magnetic field [7, 8, 9], with the Coriolis force $2m\dot{\mathbf{r}} \times \boldsymbol{\omega}$ in the former corresponding to the Lorentz force in the latter, with the identification of $-eB/c$ with $2m\omega$. Although significant insight can be gained from this connection, it is limited to the BCS regime and does not include important ingredients that are unique to a rotating superfluid. These include: the absence of screening, the centrifugal force, the trap potential, the fixed number of atoms, and the tunable resonant pairing interaction, all of which are absent in the analogous superconductor problem. In the latter a concomitant Zeeman field also appears. It can be effectively introduced into the atomic problem (but we will not do so here) by imposing a difference in the number of the two pairing atomic (hyperfine state) species [10].

In this Letter we study the effect of an imposed rotation ω on a trapped (with axially symmetric trap frequencies $\Omega_\perp$ and Ω_z and $\overline{\Omega} \equiv (\Omega_z\Omega_\perp^2)^{1/3}$) resonantly-paired superfluid near a *narrow* FR (i.e., the width of the resonance, Γ, is smaller than the Fermi energy $\epsilon_F = (3N)^{1/3}\hbar\overline{\Omega} \equiv \hbar^2 k_F^2/m$, $\gamma \equiv \sqrt{\Gamma/\epsilon_F} \ll 1$), tuned through the BEC-BCS crossover[15]. Our primary prediction is the upper-critical angular velocity $\omega_{c2}(\delta, T)$ as a function of FR detuning δ and temperature T, illustrated for $T \to 0$ in Fig. 1 and for a range of δ as a function of T in Fig. 2.

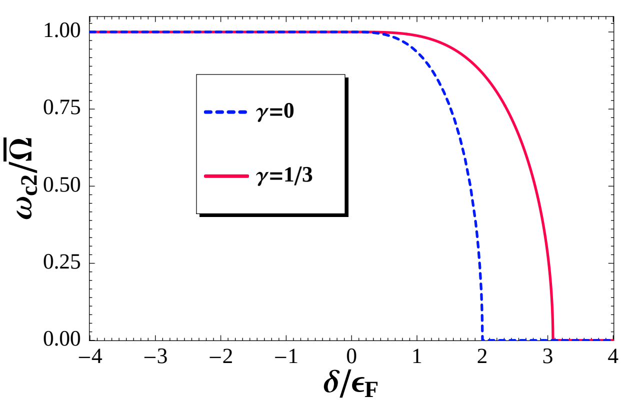

FIG. 1: (Color online) The upper critical rotation rate $\omega_{c2}(\delta, T \to 0)$ as a function of the detuning δ in an isotropic trap with 10^7 atoms, and for two FR widths $\gamma = 0, 1/3$. ω_{c2} vanishes for δ greater than $\delta_c \approx 2\epsilon_F + \frac{\gamma}{2}\epsilon_F \ln(\epsilon_F/\hbar\Omega_\perp)$.

For low T and $\Omega_z \ll \Omega_\perp$, in the BCS and crossover regimes of $0 \lesssim \delta < \delta_c$ we find ω_{c2} to be *implicitly* given by $\hbar\sqrt{\omega_{c2}^2 + \Omega_\perp^2} \approx 2\Delta_{\omega_{c2}}\sqrt{\hbar\Omega_\perp/\epsilon_F^{\omega_{c2}}}$, controlled by the discreteness of the rotated trap spectrum cutting off the weak Fermi-surface Cooper pairing characterized by the BCS gap $\Delta_{\omega=0} \equiv \Delta(\epsilon_F) = 8e^{-2}\epsilon_F \exp[-\frac{\delta - 2\epsilon_F}{\sqrt{\Gamma\epsilon_F}}]$. In this regime, for $\frac{\gamma}{2}\ln(\epsilon_F/\hbar\Omega_\perp) \gg 1$, this leads to

$$\omega_{c2}^{T=0}(\delta) \approx \begin{cases} \sqrt{6}\Omega_\perp\sqrt{1-\frac{\delta}{\delta_c}}, & \text{for } \delta \to \delta_c^-, \\ \Omega_\perp\sqrt{1-(\frac{\delta}{\delta_c})^6} & \text{for } 0 \lesssim \delta \ll \delta_c, \end{cases} \quad (1)$$

where the behavior near $\delta_c \approx 2\epsilon_F + \frac{\gamma}{2}\epsilon_F \ln(\epsilon_F/\hbar\Omega_\perp)$ corresponds to a vanishing of $T_c^{\omega=0}$ when the BCS condensation energy Δ^2/ϵ_F drops down to the trap level spacing

$\hbar\Omega_\perp$. At low detuning $\delta \ll \delta_c$, ω_{c2} rises up to but is limited below $\Omega_\perp$ by the implicit dependence entering through $\epsilon_F^\omega = \epsilon_F(1-\omega^2/\Omega_\perp^2)^{1/3}$ and $\Delta_\omega = \Delta(\epsilon_F^\omega)$ due to the centrifugal and Coriolis forces reducing the effective trap potential and atom density. In this limit, we recover the bulk result $\hbar\omega_{c2} \approx 2\Delta_{\omega_{c2}}^2/\epsilon_F^{\omega_{c2}}$, corresponding to Gorkov's (fixed chemical potential) prediction for type-II superconductors [8].

In the opposite limit of $\frac{\gamma}{2}\ln(\epsilon_F/\hbar\Omega_\perp) \ll 1$, $\delta_c \approx 2\epsilon_F$ and $\omega_{c2}^{T=0}$ reduces to

$$\omega_{c2}^{T=0}(\delta) \approx \omega_*(\delta) = \Omega_\perp\sqrt{1-(\delta/2\epsilon_F)^3}, \text{ for } 0 < \delta \lesssim 2\epsilon_F, \tag{2}$$

This also smoothly matches onto the $\omega_{c2} \to \Omega_\perp^-$ behavior in the BEC regime of $\delta < 0$, where molecular superfluidity can only be destroyed by large quantum fluctuations associated with comparable boson and vortex densities, where it undergoes transitions to a variety of bosonic quantum Hall states [11].

As can be seen in Fig. 1 and Eq. (1), a distinction between BCS ($\delta \gg 2\epsilon_F$) and crossover ($0 < \delta \lesssim 2\epsilon_F$) regimes is not reflected in $\omega_{c2}(\delta)$, controlled throughout by weak Cooper pairing. However, $T_c(\omega)$ (Fig. 2) does distinguish between these regimes, exhibiting a point of inflection at a scale $\omega_*(\delta)$ in the crossover (but not in the BCS) regime. At a rotation rate $\omega < \omega_*(\delta)$ (nonzero only in the crossover regime) a finite number of molecular bosons $N_B(\omega)$ is present and $T_c(\omega)$ is set by $T_c^{\rm BEC}[N_B(\omega,\delta)]$. At a higher rate, $\omega_*(\delta) < \omega < \omega_{c2}(\delta)$ (as a consequence of an increased atomic density of states) ϵ_F^ω drops below $\delta/2$ and $T_c(\omega)$ is determined by an exponentially reduced bulk BCS value $T_{\rm c}^{\rm BCS}(\epsilon_F^\omega) = \frac{e^c}{\pi}\Delta(\epsilon_F^\omega)$ ($c \approx 0.577$ is the Euler-Mascheroni constant).

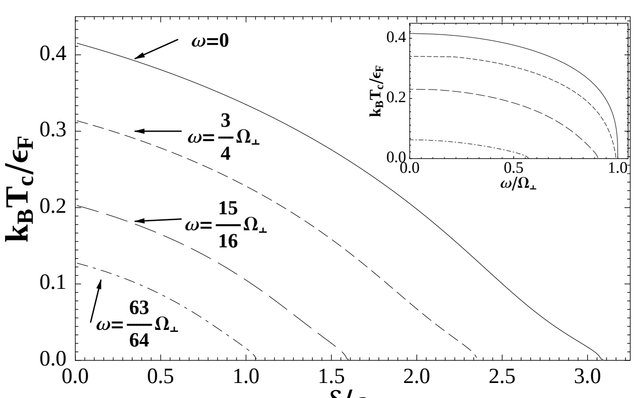

FIG. 2: Main: $T_c(\delta,\omega)$ in a spherical trap for a range of rotation rates ω as a function of δ. Inset: $T_c(\delta,\omega)$ for a range of detuning $\delta = \{0, 0.9, 1.8, 2.7\}\epsilon_F$ as a function of ω with the top line being for $\delta = 0$. In both cases, $N = 10^7$ and $\gamma = 1/3$.

In the BCS regime ($\delta > 2\epsilon_F$), at slow rotation $\omega \ll \omega_{c2}^{T=0}$, we find

$$\frac{T_c(\omega)}{T_{\rm c,bulk}^{\rm BCS}} = 1 - \frac{7\zeta(3)}{12\pi^2}\frac{\epsilon_F\hbar\left(\Omega_\perp^2+\omega^2\right)}{\Omega_\perp\left(k_B T_{\rm c,bulk}^{\rm BCS}\right)^2} + \dots, \tag{3}$$

with the trap responsible for the quadratic in ω suppression of T_c in the BCS regime, contrasting to the linear $|\omega|$ suppression in a superconductor [7]. In the BEC regime, the rotation suppresses T_c according to

$$T_c(\omega) \approx T_c^{\rm BEC}\left(1-\omega^2/\Omega_\perp^2\right)^{1/3}, \text{ for } \delta \lesssim 0. \tag{4}$$

We now sketch the derivation of these results, delaying details to a future publication [12].

A gas composed of two hyperfine species (labeled by $\sigma = \uparrow, \downarrow$) of fermionic atoms interacting through a tunable (via a "bare" detuning δ_0) s-wave FR, corresponding to a closed-channel diatomic molecular state, is well-characterized by a Hamiltonian $\mathcal{H} = \mathcal{H}_F + \mathcal{H}_B + \mathcal{H}_g$, [13, 14, 15] where

$$\mathcal{H}_F = \sum_\sigma \int d^3\mathbf{r}\, \psi_\sigma^\dagger(\mathbf{r})\left[-\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r}) - \mu - \omega L^z\right]\psi_\sigma(\mathbf{r}),$$

$$\mathcal{H}_B = \int d^3\mathbf{r}\, \phi^\dagger(\mathbf{r})\left[-\frac{\hbar^2\nabla^2}{4m} + 2V(\mathbf{r}) - 2\mu + \delta_0 - \omega L^z\right]\phi(\mathbf{r}),$$

$$\mathcal{H}_g = g\int d^3\mathbf{r}\Big(\phi^\dagger(\mathbf{r})\psi_\uparrow(\mathbf{r})\psi_\downarrow(\mathbf{r}) + \psi_\downarrow^\dagger(\mathbf{r})\psi_\uparrow^\dagger(\mathbf{r})\phi(\mathbf{r})\Big), \tag{5}$$

written in the frame rotating with $\omega\hat{z}$. Here $\psi_\sigma^\dagger(\mathbf{r})$ is an atomic creation operator in the hyperfine state $|\sigma\rangle$ whereas $\phi^\dagger(\mathbf{r})$ is the molecular creation operator. Above, m is the fermion mass, μ the chemical potential, g the atom-molecule interconversion amplitude, $L^z = -i\hbar\hat{z}\cdot(\mathbf{r}\times\nabla_{\mathbf{r}})$ the angular momentum operator, and $V(\mathbf{r}) = \frac{1}{2}m\left(\Omega_\perp^2(x^2+y^2) + \Omega_z^2 z^2\right)$ is the atomic trap potential.

To determine $\omega_{c2}(\delta, T)$ we look for an instability of the normal state to a superfluid state. To this end we consider system's partition function $Z = {\rm Tr}[e^{-\beta\mathcal{H}}]$, ($\beta = 1/k_B T$) and integrate out Fermi fields perturbatively in the FR coupling g, valid when the corresponding dimensionless coupling $\gamma = \frac{1}{4\pi^2}\frac{g^2}{\sqrt{\epsilon_F}}\left(\frac{2m}{\hbar^2}\right)^{3/2} \ll 1$. We obtain $Z = \int D\overline{\phi}D\phi\exp(-S_\phi/\hbar)$ with (defining $\hat{h}_B = -\hbar^2\nabla^2/4m + 2V(r) - 2\mu + \delta_0 - \omega L_z$),

$$S_\phi = -\hbar\ln\left[Z_F^{(0)}\right] + \int_0^{\beta\hbar} d\tau \int d^3\mathbf{r}\,\overline{\phi}(\mathbf{r},\tau)\left(\hbar\partial_\tau + \hat{h}_B\right)\phi(\mathbf{r},\tau)$$
$$+ \frac{1}{\hbar}\int_0^{\beta\hbar} d\tau d\tau' \int d^3\mathbf{r} d^3\mathbf{r}'\,\overline{\phi}(\mathbf{r},\tau)\Sigma(\mathbf{r},\mathbf{r}',\tau-\tau')\phi(\mathbf{r}',\tau'), \tag{6}$$

where $\Sigma(\mathbf{r},\mathbf{r}',\tau) = -g^2\mathcal{G}_F^{(0)}(\mathbf{r},\mathbf{r}',\tau)\mathcal{G}_F^{(0)}(\mathbf{r},\mathbf{r}',\tau)$ is the molecular (Cooper pair) self-energy arising from molecule fluctuations into a pair of atoms governed by the free fermion propagator $\mathcal{G}_F^{(0)}(\mathbf{r},\mathbf{r}',\tau) = -\langle\psi_\sigma(\mathbf{r},\tau)\overline{\psi}_\sigma(\mathbf{r}',0)\rangle_0$ and $Z_F^{(0)} = {\rm Tr}\left[e^{-\beta\mathcal{H}_F}\right]$ is the partition function of free fermions. Higher-order corrections in g lead to the inclusion of molecular interactions induced by the atomic FR, small for $\gamma \ll 1$.

The instability to the superfluid state at $\omega_{c2}(\delta, T)$ is determined by the vanishing of the lowest eigenvalue of the inverse

of the effective molecular propagator $\tilde{\mathcal{G}}_B^{-1}(\mathbf{r},\mathbf{r}';i\omega_\ell)|_{i\omega_\ell=0} \equiv \tilde{\mathcal{G}}_B^{(0)-1}(\mathbf{r},\mathbf{r}';i\omega_\ell) - \tilde{\Sigma}(\mathbf{r},\mathbf{r}',i\omega_\ell)|_{i\omega_\ell=0}$, where $\mathcal{G}_B^{(0)}(\mathbf{r},\mathbf{r}',\tau) = -\langle\phi(\mathbf{r},\tau)\overline{\phi}(\mathbf{r}',0)\rangle_0$ is the free molecular propagator and $\omega_\ell = 2\ell\pi/(\beta\hbar), (\ell \in \mathbb{Z})$ is the bosonic Matsubara frequency[16]. To lowest order in g^2, $\tilde{\mathcal{G}}_B^{-1}(\mathbf{r},\mathbf{r}';0)$ is diagonalized by eigenstates of $\tilde{\mathcal{G}}_B^{(0)-1}(\mathbf{r},\mathbf{r}';0)$ (valid for a narrow resonance), that are simply rotated trap eigenstates $\phi_\mathbf{n}(\mathbf{r})$ defined by $(-\hbar^2\nabla_\mathbf{r}^2/4m + 2V(\mathbf{r}) - \omega L_z)\phi_\mathbf{n}(\mathbf{r}) = \epsilon_\mathbf{n}^0\phi_\mathbf{n}(\mathbf{r})$, with eigenvalues ($\mathbf{n} = \{n_0, n, n_z\}$) [17]

$$\epsilon_\mathbf{n}^0 = \hbar\Omega_\perp(2n+1) - 2\hbar\omega n_z + \hbar\Omega_z(n_0 + 1/2), \tag{7}$$

where $n_0 \in \mathbb{N}, n \in \frac{1}{2}\mathbb{N}$ and $n_z = \{-n, -n+1, \ldots, n\}$ are the axial, radial and angular momentum quantum numbers. The single-particle fermionic and bosonic spectra are then given by $\epsilon_\mathbf{n}^{0(F)} = \epsilon_\mathbf{n}^0 - \mu$ and $\epsilon_\mathbf{n}^{0(B)} = \epsilon_\mathbf{n}^0 + \delta_0 - 2\mu$, respectively. In the weakly-interacting limit the condensation is therefore into $\mathbf{n} = \mathbf{0} \equiv \{0,0,0\}$, a nonrotating bosonic eigenmode localized at the center of the trap [17].

Projecting the self-energy operator $\tilde{\Sigma}(\mathbf{r},\mathbf{r}';0)$ onto the condensate $\phi_\mathbf{0}(\mathbf{r})$ gives ω_{c2} as the solution of the Thouless criterion $\epsilon_\mathbf{0}^{(B)} = \epsilon_\mathbf{0}^{0(B)} - \int d^3\mathbf{r}d^3\mathbf{r}'\phi_\mathbf{0}^*(\mathbf{r})\tilde{\Sigma}(\mathbf{r},\mathbf{r}';0)\phi_\mathbf{0}(\mathbf{r}')$, explicitly given by

$$1 = \frac{g^2}{\delta - 2\mu}\left\{\sum_{\mathbf{n},\mathbf{n}'}|R_{\mathbf{n},\mathbf{n}'}|^2\frac{\tanh\left(\beta\epsilon_\mathbf{n}^{0(F)}/2\right) + \tanh\left(\beta\epsilon_{\mathbf{n}'}^{0(F)}/2\right)}{2\left(\epsilon_\mathbf{n}^{0(F)} + \epsilon_{\mathbf{n}'}^{0(F)}\right)} - \int\frac{d^3k}{(2\pi)^3}\frac{m}{\hbar^2k^2}\right\}, \tag{8}$$

where we have expressed the "bare" detuning, δ_0, appearing in the Hamiltonian in terms of the physical detuning $\delta = \delta_0 - g^2\int\frac{d^3k}{(2\pi)^3}\frac{m}{\hbar^2k^2}$ arising in the two-body s-wave scattering measurement through the scattering length $a_s = -\frac{mg^2}{4\pi\hbar^2\delta}$. The matrix element $R_{\mathbf{n},\mathbf{n}'} \equiv 2^{-3/4}\int d^3\mathbf{r}\phi_\mathbf{0}(\mathbf{r})\phi_\mathbf{n}^*(\mathbf{r}/\sqrt{2})\phi_{\mathbf{n}'}^*(\mathbf{r}/\sqrt{2})$ is given by

$$|R_{\mathbf{n},\mathbf{n}'}| = \frac{1}{2}\left(\frac{m\overline{\Omega}}{2\pi\hbar}\right)^{3/4}\frac{[1+(-1)^{n_0+n_0'}](n_0+n_0')!}{2^{(n_0+n_0')}\left(\frac{n_0+n_0'}{2}\right)!\sqrt{n_0!n_0'!}}\frac{(n+n')!}{2^{n+n'}}\frac{\delta_{n_z+n_z',0}}{\sqrt{(n-n_z)!(n+n_z)!(n'+n_z')!(n'-n_z')!}}. \tag{9}$$

Equation (8) needs to be solved together with the total atom number equation $N = \beta^{-1}d\ln Z/d\mu = \int d^3\mathbf{r}\left(2\langle\phi^\dagger\phi\rangle + \langle\psi_\sigma^\dagger\psi_\sigma\rangle\right)$, fixing μ in terms of N, which to this order in g is given by $N = N_F^0 + 2N_B + \mathcal{O}(g^2)$ with $N_F^0 = 2\sum_\mathbf{n}1/(e^{\beta\epsilon_\mathbf{n}^{0(F)}}+1)$ and $N_B = \sum_\mathbf{n}1/(e^{\beta\epsilon_\mathbf{n}^{(B)}}-1)$.

Analytic analysis of Eq. (8) is possible due to considerable simplifications in the limits of a narrow resonance ($\gamma \ll 1$) and a Fermi energy that is large compared to the rotating trap level spacing, $\epsilon_F \gg \hbar(\Omega_\perp + \omega)$. The latter ensures that the fermionic trap level sums are dominated by large quantum numbers $\mathbf{n},\mathbf{n}'$, allowing the Gaussian approximation $(\frac{1}{2})^{n+n'}(n+n')!/(n!n'!) \approx (\pi n)^{-1/2}\exp\left[-(n-n')^2/4n\right]$. Physically this approximation corresponds to a weak Coriolis force, with the atomic trajectories nearly straight lines (locally well-characterized by plane waves) turning by a small fraction of the particle spacing, $n^{-1/3}$. This reduces the Thouless criterion, Eq. (8), to

$$\frac{\delta - 2\mu}{\gamma\sqrt{\epsilon_F\mu}} = F(\beta\mu) + G\left[\beta\mu, \beta\hbar\Omega_\perp\left(1 + \frac{\omega_{c2}^2}{\Omega_\perp^2}\right), \beta\hbar\Omega_z\right], \tag{10}$$

where

$$G(x,a,b) = \int_0^\infty\int_0^\infty dydp\int_0^1 dz\frac{e^{-p^2}\sqrt{\frac{y}{\pi x}}\tanh(\frac{y-x}{2})\frac{-1}{y-x}}{1 + \frac{\cosh^2(\frac{y-x}{2})}{\sinh^2\left(\frac{p}{2}\sqrt{y[a+(b-a)z^2]}\right)}},$$

$$G(x \gg 1, a, b) = \int_{-\infty}^\infty dp\frac{-e^{-p^2}}{\sqrt{\pi}}\int_0^1 dz\left[c_1 + \psi_0\left(\frac{1}{2} + iQ\right)\right], \tag{11}$$

$$F(x) = \int_0^\infty dy\sqrt{\frac{y}{4x}}\left[\frac{\tanh((y-x)/2)}{y-x} - \frac{1}{y}\right], \tag{12}$$

$$F(x \gg 1) = \ln(8x/\pi) - 2 + c, \tag{13}$$

with $Q = p\sqrt{x(a+(b-a)z^2)}/(2\pi)$, $c_1 = \ln 4 + c$, and $\psi_0(y)$ the digamma function. Within the same approximation, the number equation reduces to

$$N \approx 2\left(\frac{k_BT}{\hbar\overline{\Omega}[\omega]}\right)^3\left[-Li_3\left(-e^{-\beta\epsilon_\mathbf{0}^{0(F)}}\right) + Li_3\left(e^{-\beta\epsilon_\mathbf{0}^{(B)}}\right)\right], \tag{14}$$

where $Li_3(x) = \sum_{k=1}^\infty\frac{x^k}{k^3}$ is the trilogarithm function, with asymptotic forms $Li_3(-e^z) \approx -z^3/6$ for $z \gg 1$, $Li_3(-e^z) \approx -e^z$ for $z \ll -1$, and $Li_3(1) \equiv \zeta(3) \approx 1.202$. The first and second terms are the number of atoms and thermally excited molecules (with the condensate vanishing at

ω_{c2}), respectively, and $\overline{\Omega}(\omega) \equiv \overline{\Omega}(1-\omega^2/\Omega_\perp^2)^{1/3}$ is the effective trap frequency, reduced by the centrifugal "potential".

In the thermodynamic limit $\beta\hbar\Omega_{\perp,z} \ll 1$ and $\omega = 0$, $G(x,0,0) = 0$ and (for a narrow FR, $\gamma \ll 1$) we recover the BCS-BEC crossover with $T_c(\delta)$ ranging from an exponentially small BCS-regime ($\delta > 2\epsilon_F \gg k_B T$) value $k_B T_{c,\mathrm{bulk}}^{\mathrm{BCS}} = \frac{8}{\pi} e^{-2+c}\epsilon_F \exp\left[-\frac{\delta-2\epsilon_F}{\gamma\epsilon_F}\right]$ through $T_c^{\mathrm{cross.}} \approx T_c^{\mathrm{BEC}}[1-\left(\frac{\delta}{2\epsilon_F}\right)^3]^{1/3}$ in the crossover $0<\delta<2\epsilon_F$ regime and saturating at $T_c^{\mathrm{BEC}} = (2\zeta(3))^{-1/3}\hbar\overline{\Omega}N^{1/3}$ for large negative δ.

At finite ω and low T, $\beta\Omega_\perp \gg 1$ (specializing for simplicity to an anisotropic trap $\beta\Omega_\perp \gg 1$, $\beta\Omega_z \ll 1$), using $G(x\gg 1, a\gg 1, b\ll 1) \approx 1-\frac{1}{2}c+\frac{1}{2}\ln\left(\frac{\pi^2}{4xa}\right)$, we find an *implicit* equation $\hbar\sqrt{\omega_{c2}^2+\Omega_\perp^2} = \frac{1}{2}e^{1+c/2}\Delta_{\omega_{c2}}\sqrt{\hbar\Omega_\perp/\epsilon_F^{\omega_{c2}}}$ quoted in the introduction. In the BCS and crossover regimes the implicit ω_{c2} dependence enters through μ as the solution of the number equation, Eq. (14), that gives $\mu \approx \epsilon_F^\omega \approx (3N)^{1/3}\hbar\overline{\Omega}[\omega]$. The resulting solution for ω_{c2} is illustrated in Fig. 1, with asymptotics, controlled by a dimensionless parameter $\frac{\gamma}{2}\ln(\epsilon_F/\hbar\Omega_\perp)$, summarized in Eqs.(1, 2).

We find that ω_{c2} is driven to zero for $\delta > \delta_c \approx 2\epsilon_F + \frac{\gamma}{2}\epsilon_F\ln(\epsilon_F/\hbar\Omega_\perp)$. This corresponds to a vanishing of $T_c^{\omega=0}$ when the BCS condensation energy Δ^2/ϵ_F becomes comparable to trap level spacing $\hbar\Omega_\perp$. Equivalently, this condition corresponds to an oscillator length $a_\perp = \sqrt{\hbar/m\Omega_\perp}$ dropping to the coherence length $\xi = \hbar v_F/\pi\Delta$.

In the opposite limit of low detuning, $\omega_{c2}(\delta \ll \delta_c) \to \Omega_\perp^-$, where the centrifugal and trapping potentials nearly cancel, the system becomes translationally invariant and we obtain the bulk result $\hbar\omega_{c2} \approx 8^{-1}e^{2+c}\Delta_{\omega_{c2}}^2/\epsilon_F^{\omega_{c2}}$, corresponding to Gorkov's (fixed chemical potential) prediction for type-II superconductors [8]. This matches onto the BEC regime $\omega_{c2}(\delta<0) \approx \Omega_\perp$, where a purely molecular superfluid can only be destroyed by a sufficient dilution down to comparable boson and vortex densities, driving a quantum transition into a variety of bosonic quantum Hall states [11].

At high T (near T_c^0) and slow rotation ($\omega \ll \omega_{c2}^{\mathrm{T}=0}$) in the BCS regime we expand $G(x\gg 1, a, 0)$ in the limit $ax \equiv \beta^2\mu\hbar\Omega_\perp(1+\omega^2/\Omega_\perp^2) \ll 1$ (remaining in the degenerate limit, $\beta\mu = x \gg 1$), obtaining a quadratic suppression of T_c with rotation and trap frequencies given in Eq. (3). In the homogeneous limit $\omega \to \Omega_\perp^-$, this reduces to $T_c(\omega)/T_{c,\mathrm{bulk}}^{\mathrm{BCS}} \approx 1-\frac{7\zeta(3)}{6\pi^2}\hbar|\omega|\mu/(k_B T_{\mathrm{Bulk}}^{\mathrm{BCS}})^2$, giving a linear reduction with $|\omega|$, as expected from Abrikosov's theory of $H_{c2}(T)$ [7].

In the crossover regime at slow rotation, $\mu \approx \delta/2 < \epsilon_F^\omega$ and a finite fraction of the atomic Fermi sea is bound into a molecular superfluid, giving $N_F(\omega,\delta) \approx (\delta/2\epsilon_F^\omega)^3 N = (1/3)(\delta/2\hbar\overline{\Omega})^3/[1-(\omega/\Omega_\perp)^2] < N$. Because the atomic density of states increases with ω according to $1/\overline{\Omega}[\omega]^3$ (heading toward the extensive degeneracy of Landau levels in the $\omega \to \Omega_\perp$ limit), at sufficiently large ω, ϵ_F^ω drops below $\delta/2$. The crossover frequency $\omega_*(\delta)$, defined by $\epsilon_F^{\omega_*} = \delta/2$, corresponds (for a narrow resonance $\gamma \ll 1$) to a vanishing molecular condensate, $N_F(\omega_*,\delta) = N$ and is given by Eq. (2). $\omega_*(\delta)$ is a lower bound for $\omega_{c2}(\delta)$ and marks the crossover between a strongly-paired molecular superfluid, with $T_c \approx T_c^{\mathrm{BEC}}[N_B(\omega,\delta)]$

$$T_c(\omega,\delta) \approx T_c^{\mathrm{BEC}}\left[1-\left(\frac{\delta}{2\epsilon_F}\right)^3-\frac{\omega^2}{\Omega_\perp^2}\right]^{1/3}, \text{ for } \omega < \omega_*(\delta), \tag{15}$$

and a weakly-paired BCS superfluid with $T_c \approx T_c^{\mathrm{BCS}}(\epsilon_F^\omega)$ for $\omega_*(\delta) < \omega < \omega_{c2}(\delta)$. For $\delta < 0$ all atoms bind into molecules, $N_F(\mu<0)$ is exponentially small, and the above result crosses over to the BEC value, $T_c(\omega,0)$, Eq. (4), quoted in the introduction. The crossover at $\omega \approx \omega_*(\delta)$ (or equivalently at $\delta \approx 2\epsilon_F^{\omega_*}$) can be seen in $T_c(\delta,\omega)$ displayed for a full range of detunings and rotation rates in Fig. 2.

To summarize, we have studied a resonantly-interacting atomic Fermi gas across a FR and computed $\omega_{c2}(\delta,T)$ below which the gas becomes unstable to a rotating paired superfluid.

We thank N. Cooper and V. Galitski for discussions. We acknowledge support from NSF DMR-04499521 (VG), DMR-0321848 (MYV, DES, LR).

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[17] Naively, this is expected to be accurate only for $\gamma\epsilon_F/\hbar\Omega_\perp \ll 1$. However, in the opposite (thermodynamic) limit of a shallow trap, the eigenfunctions of $\tilde{\Sigma}(\mathbf{r},\mathbf{r}';\omega_\ell)$ and $\hat{h}_B$ (Landau level orbitals) become identical, guaranteeing that such projection in fact gives the exact eigenvalue of $\tilde{\mathcal{G}}_B^{-1}$. Consequently, we expect this approximation to be accurate as long as a far less restrictive condition, $\gamma \ll 1$, is satisfied.